NOTES ON CALCULATION OF THE SEVIRI BAND IR3.9 REFLECTIVITY

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Abstract

The spectral range between approximately 3.5 to 4.0 µm has been used operationally by weather satellites since the end of 1970’s, starting with the launch of the AVHRR instrument aboard TIROS-N (1978). Already the early studies have indicated the potential of this spectral range for cloud top microphysical studies, involving both, reflectivity and emissivity properties of cloud particles. To calculate these, it was necessary to separate the two components (solar reflected and thermal emitted ones) contributing to the total radiance in this band during the daytime. The first methods at the end of 1980’s utilized the AVHRR band 4 (10.8 µm) to calculate the theoretical thermal component of the AVHRR band 3 (3.7 µm), and consequently the cloud top reflectivity and emissivity. Since the AVHRR 3.7 µm band is situated in the atmospheric window range with negligible absorption by atmospheric gases only, there was no need to consider absorption/emission effects by then.

The situation began to change with introduction of the 3.9 µm band aboard GOES I-M geostationary satellites since mid-1990’s. The 3.9 µm band of this series of satellites is located more to the longer wavelengths as compared to the AVHRR instrument, which results in possible influence of CO₂ absorption on the band radiance. However, due to the lack of dedicated CO₂ band within the GOES I-M imager, the scheme for retrieving reflectance in its 3.9 µm band remains principally the same as that for the AVHRR instrument.

Even more significant is the CO₂ absorption effect for the MSG SEVIRI IR3.9 band, which ranges even further to the longer wavelengths, up to about 4.3 µm. For this reason it was necessary to modify the original methods for calculating reflectivity of the 3.9 µm band, considering the CO₂ absorption effects. Possibly also some H₂O absorption can be observed in this band.

Various semi-empirical formulas for calculating the MSG IR3.9 band reflectivity have been proposed and used since the MSG launch in 2002, differing in details, but resulting in significant variations of derived output values. In the first part, this paper brings a physically-based step-by-step derivation of a general algorithm for the reflectivity calculation, including the absorption. Assumptions of the derivation are discussed and limitations of the method are shown. An overview of possible approximations is also presented, as well as a comparison to other presently used algorithms. In the second part, the paper attempts to estimate the empirical parameters, introduced in the theoretical part.

NOTING CONVENTION

θ ... solar zenith angle
λ ... satellite zenith angle
ESD ... Earth-Sun distance
SOL ... solar constant for selected channel
M ... measured radiation in the selected channel
BT ... brightness temperature in the selected channel
B(T) ... thermal radiation in the selected channel of a blackbody with a temperature T
τₛ ... transmissivity of the atmosphere in the selected channel on the path from Sun to the Earth surface / top of a cloud
τᵣ ... transmissivity of the atmosphere in the selected channel on the path from the Earth
surface / top of a cloud to the satellite

$T_{\text{top}}$ ... thermal radiation on top of the atmosphere

$T_{\text{sur}}$ ... thermal radiation at the Earth surface / top of a cloud

$R_{\text{top}}$ ... reflected solar radiation on top of the atmosphere

$R_{\text{sur}}$ ... reflected solar radiation at the Earth surface / top of a cloud

$S_{\text{top}}$ ... incoming solar radiation on top of the atmosphere

$S_{\text{sur}}$ ... incoming solar radiation at the Earth surface / top of a cloud

$\alpha$ ... absorptivity in the selected channel

$\rho$ ... reflectivity in the selected channel

$\varepsilon$ ... emissivity in the selected channel

$\tau$ ... transmissivity in the selected channel

The particular channel is identified by its reference wavelength (in micrometers) as a bottom index, e.g. $\rho_{3.9}$ stands for the reflectivity in channel IR3.9, or $B_{13.4}(T)$ stands for the thermal radiation in channel IR13.4 of a blackbody at a temperature $T$.

![Figure 1: Spatial arrangement of the solar incident and reflected radiance and the thermal emission.](image)

**THEORY**

If we want to derive $\rho$, then these relations need to be taken into account:

1. $\rho = \frac{R_{\text{sur}}}{S_{\text{sur}}}$
2. $S_{\text{sur}} = S_{\text{top}} \cdot \tau_s$
3. $R_{\text{top}} = R_{\text{sur}} \cdot \tau_R$
4. $M = R_{\text{top}} + T_{\text{top}}$
5. $T_{\text{top}} = T_{\text{sur}} \cdot \tau_R$

If the emitting surface has the temperature $T$ and emissivity $\varepsilon$, then its thermal radiation is described by the formula:

6. $T_{\text{sur}} = \varepsilon \cdot B(T)$

Putting (1), (3), (4), (5) and (6) together, we obtain:
For radiative processes, the law of conservation of energy can be written in the form (incident radiation can be either reflected, transmitted or absorbed):

\[ \rho + \tau + \alpha = 1 \]  

Assumption (X1): The emitting surface is in thermal equilibrium with its surroundings. Consequence of (X1) is the Kirchhoff's law (absorbed radiation is emitted again):

\[ \alpha = \varepsilon \]  

(By the way, the Planck's formula for a blackbody radiation is also derived under assumption of thermal equilibrium.)

Assumption (X2): The emitting surface is the Earth surface or a dense cloud. In the case of (X2) there is no transmission or it is negligible:

\[ \tau = 0 \]  

Applying (9) and (10) in (8) we get:

\[ \rho + \varepsilon = 1 \]  

Finally (11) can be applied in (7) and we obtain \( \rho \):

\[ \rho = \frac{M - \varepsilon \cdot B(T) \cdot \tau_k}{S_{sw} \cdot \tau_k} \]  

Formula (A) is a general equation, applicable for all channels, solar and thermal, and both day and night. For thermal channels \( S_{sw} = 0 \) and the formula (A) reduces to:

\[ \rho = 1 - \frac{M}{\tau_k \cdot B(T)} \]  

For solar channels \( B(T) = 0 \) and the formula (A) reduces to:

\[ \rho = \frac{M}{\tau_k \cdot S_{sw}} \]  

For the transition region around 4 µm both solar and thermal radiation need to be taken into account and the general form of the equation (A) has to be used.

Assumption (X3): The temperature \( T \) of the emitting surface can be approximated by the brightness temperature of channel IR10.8.

Since the real temperature \( T \) of the emitting surface is unknown and cannot be derived from satellite data without additional assumptions, the best solution is to approximate it with the brightness temperature of a thermal channel, in which the emissivity is close to one for all surfaces and the atmospheric absorption is low. Generally this is the atmospheric window channel close to 11 µm, used on all meteorological satellites. On MSG satellites it is channel IR10.8. That means:

\[ T \approx BT_{10.8} \]

Assumption (X4): The whole pixel is homogeneously covered by one type of surface or cloud. For non-homogenously covered pixels the resulting radiation (both solar reflected and thermal) is a combination of contributions from all surface types in the pixel area. Since both the surface type (its radiometrical characteristics) and the fractional part of each surface type are unknown, the only solution is to assume (X4). Only in that case the radiometrical characteristics of the surface can be derived from measurement.
$S_{\text{surf}}$ is given by (2) so next we need to estimate $S_{\text{top}}$. $S_{\text{top}}$ is the radiance at top of the atmosphere through a plane parallel to the Earth surface, hence the solar constant $\text{SOL}$ is the radiance through a plane normal to the direction to the Sun and at the distance $1$ AU from the Sun. So from geometry we get:

$$S_{\text{top}} = \frac{\text{SOL} \cdot \cos(\theta)}{\text{ESD}^2} \tag{13}$$

The Earth-Sun distance $\text{ESD}$ varies during the year from $0.983$ AU in perihelion to $1.017$ AU in aphelion. That means a variation of $S_{\text{top}}$ (for a fix Sun zenith angle) of about ±3 %. In first approximation this can be neglected and (13) reduces to:

$$S_{\text{top}} = \frac{\text{SOL} \cdot \cos(\theta)}{\text{ESD}^2} \tag{13a}$$

Specifically, for channel IR3.9 the solar constant has the value $\text{SOL}_{3.9} = 4.92 \text{ mW} \cdot \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{(cm}^{-1})^{-1}$, so we get from (13):

$$S_{\text{top,3.9}} = \frac{4.92 \cdot \cos(\theta)}{\text{ESD}^2} \tag{14}$$

Generally the propagation of radiation in the atmosphere in a selected direction is described by the equation of radiative transfer. In this equation there are three main terms with the following physical meaning:

a) decrement of incoming radiation due to absorption and scattering
b) thermal emission along the path
c) increment of radiation due to scattering from other directions to the selected one.

Assumption (X5): Both thermal emission of the atmosphere itself and atmospheric scattering can be neglected.

Then the equation of radiative transformation reduces to a simple form (Beer's law):

$$dI = -I \cdot \beta(s) \cdot ds \tag{15}$$

where $I$ is the intensity of radiation and $\beta(s)$ is the volume extinction coefficient as a function of the coordinate $s$. The intensity $I$ is then given by integration of equation (15) along the path (Bouguer's Law):

$$I = I_0 \ast \exp \left[-\int \beta(s) \cdot ds\right] \tag{16}$$

Integration in a general direction can be transformed to integration in the vertical direction (Figure 2a):

$$\frac{dz}{ds} = \cos(\alpha) \tag{17}$$

So for the transmissivity of the atmosphere we get:

$$\tau = \frac{I}{I_0} = \exp \left[-\int \frac{\beta(z) \cdot dz}{\cos(\alpha)}\right] \tag{18}$$
Now let us define optical thickness of the atmosphere $L$ with the prescription:

$$\int \beta(z) \, dz = \frac{d}{L}$$

(19)

where $d$ is the geometrical thickness of the atmosphere. Then we get for the transmissivity:

$$\tau = \exp \left[ -\frac{d}{L \cdot \cos(\alpha)} \right]$$

(20)

From geometry we get (Figure 2b):

$$d = l_s \cdot \cos(\theta) = l_r \cdot \cos(\lambda)$$

(21)

Applying this for the incoming and outgoing radiation we get:

$$\tau_s = \exp \left[ -\frac{l_s}{L} \right]$$

(22)

$$\tau_r = \exp \left[ -\frac{l_r}{L} \right]$$

(23)

Putting (22), (21) and (23) together, we obtain:

$$\tau_s = \left( \tau_r \right)^{\frac{\cos(\lambda)}{\cos(\theta)}}$$

(24)

If the absorption is not so strong, we can use an approximation:

$$\tau_r = 1 \Rightarrow \ln \left( \tau_r \right) = \tau_r - 1$$

(25)

Which gives:

$$\tau_s = \exp \left[ \left( \tau_r - 1 \right) \frac{\cos(\lambda)}{\cos(\theta)} \right]$$

Using (14) and (24) in (2) we get for channel IR3.9:

$$B \quad S_{\text{sur},3.9} = \frac{4.92 \cdot \cos(\theta)}{\text{ESD}^2} \cdot \left( \tau_{R,3.9} \right)^{\frac{\cos(\lambda)}{\cos(\theta)}}$$

FORMULA FOR $\tau_{R,3.9}$

The main absorbent in channel IR3.9 is CO$_2$ (carbon dioxide), but there is also slight absorption by H$_2$O (water vapour) and CH$_4$ (methane) and maybe some other trace gases. Also in other infrared channels absorption of the two main atmospheric absorbents, CO$_2$ and H$_2$O, needs to be taken into account. The absorption of each of these gases is independent on the other ones, so the transmissivity of a selected channel should have the form:

$$\tau = \tau_{(CO_2)} \cdot \tau_{(H_2O)} \cdot \tau_{(X)}$$

(26a)

Here $\tau_{(CO_2)}$ and $\tau_{(H_2O)}$ is the transmissivity of a selected channel due to CO$_2$ respectively H$_2$O absorption. The last term represents the influence of all other possible absorbents. If the selected channel is not influenced by some absorbent, the corresponding term is supposed to be equally one. For channel IR3.9 at least the first term needs to be taken into account, but possibly also the second one. So how to estimate these terms for channel IR3.9? Here an empirical formula has to be used.

So let’s try to derive a model for the $\tau_{R,3.9}$. We need to use some channel with CO$_2$ and/or H$_2$O absorption, but this absorption should not be too strong as we need (according with assumption (X5)) the main part of the radiation to originate on the surface/top of cloud. But we also need all surface types to behave more or less as a black body in this channel. The only SEVIRI channels that match...
both these requirements are IR12.0 and IR13.4. In channel IR12.0 there is H\textsubscript{2}O absorption, in channel IR13.4 there is both H\textsubscript{2}O and CO\textsubscript{2} absorption. (Then we also need to use channel IR10.8 as a reference channel without any absorption.)

Applying (C1) on channels IR3.9, IR12.0 and IR13.4 we have:

(25a) \[ \tau_{R,3.9} = \tau_{R,3.9}(\text{CO}_2) \ast \tau_{R,3.9}(\text{H}_2\text{O}) \]
(25b) \[ \tau_{R,12.0} = \tau_{R,12.0}(\text{H}_2\text{O}) \]
(25c) \[ \tau_{R,13.4} = \tau_{R,13.4}(\text{CO}_2) \ast \tau_{R,13.4}(\text{H}_2\text{O}) \]

For the transmission coefficient in a selected channel we have formula (23). For two different channels X and Y and for absorbent Z it gives:

(26a) \[ \tau_{R,X(Z)} = \exp\left(-\frac{L_y}{L_{X(Z)}}\right) \]
(26b) \[ \tau_{R,Y(Z)} = \exp\left(-\frac{L_y}{L_{Y(Z)}}\right) \]

Now what is the relation between optical thicknesses in channels X and Y? From the definition (19) we have for these channels (and for absorbent Z):

(27a) \[ \int \beta_{x(z)}(z) \cdot dz = \frac{d}{L_{X(Z)}} \]
(27b) \[ \int \beta_{y(z)}(z) \cdot dz = \frac{d}{L_{Y(Z)}} \]

Let us introduce the mass extinction coefficient $\beta'$:

(28) \[ \beta(z) = \beta' \cdot \rho(z) \]

where $\rho(z)$ is the concentration of the absorbing gas as a function of the coordinate z. Specifically for channels X and Y we have:

(28a) \[ \beta_{x(z)}(z) = \beta'_{x(z)} \cdot \rho(z) \]
(28b) \[ \beta_{y(z)}(z) = \beta'_{y(z)} \cdot \rho(z) \]

As $\beta'$ is only dependent on the wavelength, we can define a new constant $c$ with the formula:

(29) \[ \beta'_{x(z)} = c \cdot \beta'_{y(z)} \]

Combining (27a), (28a), (29), (28b) and (27b) we obtain:

(30) \[ \frac{d}{L_{X(Z)}} = \int \beta_{x(z)}(z) \cdot dz = \int \beta'_{x(z)} \cdot \rho(z) \cdot dz = \int c \cdot \beta'_{y(z)} \cdot \rho(z) \cdot dz = c \cdot \int \beta'_{y(z)} \cdot dz = c \cdot \frac{d}{L_{Y(Z)}} \]

So the result is:

(31) \[ L_{Y(Z)} = c \cdot L_{X(Z)} \]

Using (31) and (26b) in (26a) we get:

(32) \[ \tau_{R,X(Z)} = \left(\tau_{R,Y(Z)}\right)^c \]

Using this formula we can define coefficients e, f and g:
Now we can apply (32a-c) and (25b-c) in (25a):

\[ \tau_{R,3.9(CO_2)} = \left( \tau_{R,13.4(CO_2)} \right)^e \]
\[ \tau_{R,13.4(H,O)} = \left( \tau_{R,12.0(H,O)} \right)^f \]
\[ \tau_{R,3.9(H,O)} = \left( \tau_{R,12.0(H,O)} \right)^g \]

So if we introduce a new coefficient \( h = g - e \cdot f \), we finally get:

\[ \tau_{R,3.9} = \left( \tau_{R,13.4} \right)^e \cdot \left( \tau_{R,12.0} \right)^h \]

Note, that if H\(_2\)O absorption is neglected in channel IR3.9, then it means \( g = 0 \), but the form of expression (C4) remains the same.

If we assume (X3), that the BT\(_{10.8} \) is the "real" temperature of the Earth surface / top of a cloud, then we can use it to calculate "real" radiation in channels IR13.4 and IR12.0; the ratio of the measured and calculated radiance gives then the transmission coefficients for these channels.

\[ \tau_{R,13.4} = \frac{M_{13.4}}{B_{13.4}(BT_{10.8})} = \frac{B_{13.4}(BT_{11.4})}{B_{13.4}(BT_{10.8})} \]
\[ \tau_{R,12.0} = \frac{M_{12.0}}{B_{12.0}(BT_{10.8})} = \frac{B_{12.0}(BT_{12.0})}{B_{12.0}(BT_{10.8})} \]

In real images, the value of BT\(_{13.4} \) is from the interval 180 K < BT\(_{13.4} \) < 300 K and the value of BT\(_{12.0} \) is from the interval 180 K < BT\(_{12.0} \) < 340 K. Let us assume, that the functions B\(_{13.4}(T) \) and B\(_{12.0}(T) \) can be approximated within this interval as a power function.

\[ B_{13.4}(T) \approx i \cdot T^j \]
\[ B_{12.0}(T) \approx k \cdot T^l \]

Then we can write:

\[ \tau_{R,13.4} = \frac{M_{13.4}}{B_{13.4}(BT_{10.8})} = \frac{B_{13.4}(BT_{13.4})}{B_{13.4}(BT_{10.8})} = \left( \frac{BT_{13.4}}{BT_{10.8}} \right)^j \]
\[ \tau_{R,12.0} = \frac{M_{12.0}}{B_{12.0}(BT_{10.8})} = \frac{B_{12.0}(BT_{12.0})}{B_{12.0}(BT_{10.8})} = \left( \frac{BT_{12.0}}{BT_{10.8}} \right)^l \]

If we combine (C4) and (36a-b) and introduce new parameters M = j \cdot e and N = l \cdot h, we get the following formula:

\[ \tau_{R,3.9} = \left( \frac{BT_{13.4}}{BT_{10.8}} \right)^M \cdot \left( \frac{BT_{12.0}}{BT_{10.8}} \right)^N \]

where the values of M and N can be determined empirically.

**DERIVATION OF EMPIRICAL PARAMETERS**

To determine the empirical parameters introduced in the theoretical part, following procedure was used:
To avoid possible problems with determining the incoming solar radiation (like occurrence of Sun glint), midnight images were used, so only thermal radiation is present in the images.

For night conditions equation (A1) can be used to obtain relation between surface reflectivity and transmissivity of the atmosphere in channel IR3.9 (together with assumption (X3)). That means that we need to know the surface reflectivity for a given pixel first. Water surface was chosen as a surface with known reflectivity. The reflectivity is supposed to be zero (or very close to zero). This surface was also chosen, because it can be identified quite easy in the images.

Cloud free pixels with water surface were identified manually (using human eyes to be more exact) using standard RGB composites 'Night Microphysical' and '24-hour Microphysical' (as recommended by EUMETSAT in the MSG Interpretation Guide). An additional criterion $BT_{10.8} > +2^\circ C$ was used to exclude pixels with possible coverage by sea ice (or at least most of them).

Four (more or less randomly chosen) images were used, in each about 80 pixels were selected, covering more or less homogeneously all seas (and some lakes) visible on the Earth disc. Total number of 330 pixels was used. For each of these pixels brightness temperatures in the infrared channels were read and with these values further calculations were done.

Equation (34a-b) was used to calculate transmissivity of channels IR13.4 and IR12.0 and equation (36a-b) to obtain parameters $j$ and $l$ (this was a proof that approximation (35a-b) can be used). Regression gives the values $j = 3.96$ and $l = 4.23$ as the best results.

Equation (A1) with $\rho = 0$ was used to calculate transmissivity in channel IR3.9. At first look there does not seem to be any dependence between transmissivities in channels IR13.4 and IR3.9. That means that parameter N is not zero. Also the transmissivity in channel IR3.9 seems to vary with latitude. That is easy to explain as the water vapour content in the atmosphere also varies with latitude.

Finally the regression model (C5) was used. The values $M = 3.28$ and $N = -21.92$ were obtained. It is obvious that these values are not fully validated. They should be considered as a first suggestion.

**SUMMARY**

To summarize, the final formulas derived above are as follows:

\[(A) \quad \rho_{3.9} = \frac{M_{3.9}}{S_{\text{sur},3.9} - B_{3.9}(BT_{10.8})} \]

\[(B) \quad S_{\text{sur},3.9} = \frac{4.92 \cos(\theta)}{ESD^2} \sin(\theta)^{4} \]

\[(C) \quad \tau_{R,3.9} = \left(\frac{BT_{13.4}}{BT_{10.8}}\right)^{3.28} \left(\frac{BT_{12.0}}{BT_{10.8}}\right)^{-21.92} \]

These three equations together can be used in operational applications. The first two are physically based and can be used universally, the third one is a semiempirical formula and the exponents can vary a little. This paper should be considered as a description of the methodology, rather than bringing final results. Further validation of formula (C) would be desirable.