Estimation of ASCAT-Normalised Radar Cross Section: ATBD

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v.1	23 July 2014		Added explanation in Section 5.2. Addressed all comments after first technical review.
v.2	24 July 2014		Changed signature list. Updated document title.

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1 INTRODUCTION

1.1 Purpose and Scope of Document

The Advanced Scatterometer (ASCAT) is a radar system carried on board the METOP series of satellites. The primary objective of ASCAT is to measure ocean backscatter for the retrieval of ocean surface wind vectors. This document gives an overview of the ASCAT instrument and the algorithms used to convert its measurements into calibrated and geolocated Normalised Radar Cross Section (NRCS, also referred to as σ_0).

1.2 Structure of the Document

The document is organised in the following sections:

Section 1	This introduction
Section 2	Describes the ASCAT instrument.
Section 3	Models the ASCAT measurement process.
Section 4	Gives an overview of ASCAT external calibration.
Section 5	Describes the ground processing of ASCAT data.
Section 6	Presents a summary.
Appendix A	Describes the geolocation algorithm.
Appendix B	Details the algorithm for calculating *Kp*.

1.3 Acronyms Used in this Document

Acronym	*Meaning*
ADC	Analog-to-Digital Converter
ANTRF	Antenna assembly hosting the right fore and left aft antennas
ANTM	Antenna assembly hosting the left and right mid antennas
ANTRA	Antenna assembly hosting the right aft and left fore antennas
ANX	Ascending Crossing Node
ASCAT	Advanced SCATterometer
DPU	Digital Processing Unit
ERS AMI	European Remote Sensing (satellite) Active Microwave Instrument
EUMETSAT	European Organisation for the Operation and Exploitation of Meteorological Satellites
FM	Frequency Modulated
FPM	Fine Pointing Mode
HPA	High Power Amplifier
ISP	Instrument Source Packet
LNA	Low Noise Amplifier
METOP	Meteorological Operational Platform
NRCS	Normalised Radar Cross-Section

Acronym	*Meaning*
OSV	Orbit State Vector
PG	Power Gain
PRI	Pulse Repetition Interval
RCS	Radar Cross Section
RF	Radio Frequency
RFU	Radio Frequency Unit
SFE	Scatterometer Front End
SLR	Side Looking Radar
TPR	Transmission Power Ratio
YSM	Yaw Steering Mode

1.4 Reference Documents

	Document Name	*Reference Number*
RD 1	*Quick computation of the distance between a point and an ellipse* L. Maisonobe (2006)	

2 DESCRIPTION OF THE ASCAT INSTRUMENT

2.1 Overview

ASCAT has six antennas which operate at C band with vertical polarisation. Figure 1 shows the viewing geometry of ASCAT. Three of the antennas look at a swath on the left hand side of the sub-satellite track and the others look at a swath on the right hand side.

A linear frequency modulated (FM) pulse is generated by the instrument and transmitted by one of the antennas. The received echo is “dechirped” and Fourier-transformed to produce a power spectrum as a function of range. This is then averaged along track and transmitted to the ground along with ancillary data. The on-board processing does not require knowledge of satellite position or attitude and gives a substantial reduction of the data rate. The open loop gain of the instrument is monitored by an internal calibration subsystem.

In the ground processor, corrections are made for variations in the instrument gain and the receiver noise and filter shape. Normalisation factors derived from the known antenna gain pattern are used to convert the corrected power into absolutely calibrated backscatter. Spatial averaging at specific points on the Earth's surface produces collocated backscatter measurements from the three antenna beams, which can be used for the retrieval of ocean wind vectors. An external calibration procedure using ground based transponders is used to estimate the antenna gain patterns.

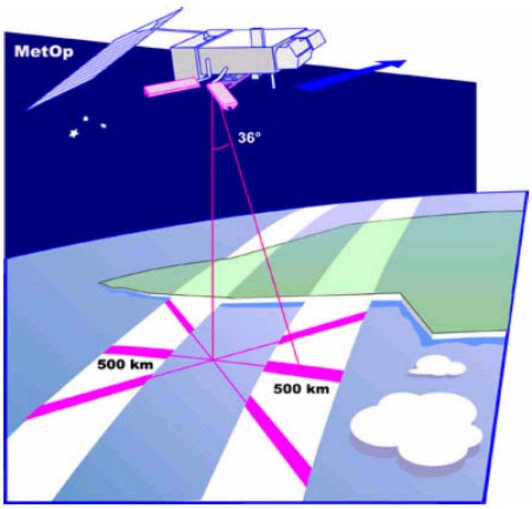

Figure 1: Viewing geometry of the ASCAT.

2.2 Physical Description

The ASCAT instrument consists of the following components:

- Three antenna assemblies each of which hosts two antennas. The ANTRF assembly contains the right fore and left aft antennas, the ANTM assembly contains the left and right mid antennas, the ANTRA assembly contains the right aft and left fore antennas.
- The scatterometer front end sub system (SFE), containing the switching matrix used for switching the RF electronics between the six antennas, the low noise amplifier (LNA) for the preamplication of the echo and the internal calibration signal, the calibration signal couplers and detectors used for internal calibration, filters implemented in the transmitter and receiver paths to reduce harmonic emissions and susceptibility to out of band signals.
- The radio frequency (RF) electronics, including the radio frequency unit (RFU) which performs the RF chirp generation and the down conversion and deramping of the received signal, a high power amplifier (HPA) for the amplification of the transmit signal.
- A digital processing unit (DPU) performing the analog to digital conversion of the received signal, the power spectrum estimation, instrument time lining and the data formatting and packetising.
- An instrument control unit which performs the command handling, acquisition and management of housekeeping data and redundancy management.
- The power distribution unit.

In deployed configuration, the three antenna assemblies are pointed towards the Earth and oriented to provide three different azimuth angles (45° for ANTRA, 90° for ANTRM and 135° for ANTRF with respect to the satellite x axis).

The two side antennas (which are deployed at the beginning of the satellite operation) are on the same side of the satellite. The mid-antenna assembly (which is fixed) is mounted on the Earth-facing panel.

2.3 Modes of Operation

The ASCAT has two main modes of operation:

- Measurement mode: In this mode, all six antennas are activated in sequence. Echo, internal calibration and noise samples are recorded from each antenna.
- External calibration mode: Similar to measurement mode, except that two antennas are operated in sequence. The first of these (the “calibration" antenna) illuminates a ground based transponder while the second antenna (the “disposal" antenna) is used to keep the duty cycle the same as in measurement mode.

2.4 Internal Calibration

ASCAT has a number of internal calibration systems which provide ancillary data for use in the ground processing to compensate for the following:

- drifts in the transmit gain, receive gain or RF path losses,
- drifts in the transmission power reflected from the antenna,
- filter ripples and noise in the measurement data.

Figure 2 shows the timeline of the nominal transmit/receive cycle. It has four phases: high power signal transmission, echo reception, internal calibration and noise measurement. The transmit signal is partially reflected by the antenna. In order to take this into account the transmit power and the reflected power of every pulse are measured by the forward and the reflected power detectors. This allows the estimation of the transmission power ratio (TPR) which is then used within the ground processing to correct the nominal measurement data. The characterisation of the forward and reflected power detectors (in terms of power versus detector output voltages and detector temperature) is performed on ground before launch.

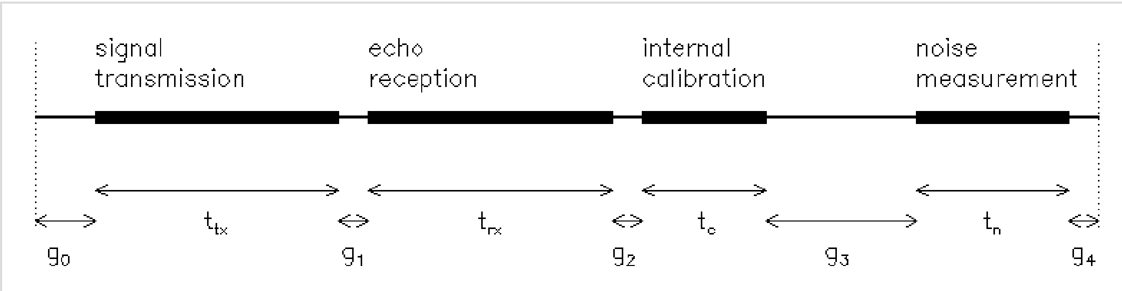

Figure 2: ASCAT transmit/receive timeline.

During the internal calibration phase, a signal is generated by the RFU which is measured at the forward power detector and at the main ADC. The power of the signal is regulated so that it is the same as the transmit power reading. It experiences the same gain or loss variation as the nominal measurement data and therefore has a signal level which is also proportional to the receive gain. The power level of the calibration signal at the ADC is proportional to the product of transmit power P and receive gain G.

The noise measurements are the last phase of each transmit/receive cycle and at a time when all echoes have died away. The ASCAT DPU samples the noise signal which originates mainly in the low noise amplifier but which could also come from outside the instrument (e.g. possible emission from other instruments or thermal radiation from the ground surface). The noise samples are processed in a similar manner to the measurement data.

The ASCAT internal calibration gives the PG product only at one fixed frequency. Gain variations (filter ripples) across the full ASCAT bandwidth are estimated from averaged noise data. In the ground processing the noise measurements from all six antennas over a time span of typically 10 minutes are averaged to get the mean noise power versus frequency with sufficient resolution. This is then normalised with respect to the calibration frequency to obtain the actual filter shape.

The measured signal is the sum of the echo signal and noise. After compensation of the filter shape all samples of the signal have the same noise floor. It is therefore possible to average over the noise samples to get one “mean noise floor" value which is subtracted from the measured signal. Due to the averaging over discriminator frequency, it is sufficient to use a smaller number of noise measurements (from typically 2.5 minutes). The noise power may be different for each antenna and is therefore calculated individually.

2.5 Instrument Source Packets

The ASCAT DPU produces a stream of packets which are stored and periodically transmitted to a ground station. These contain either the power spectrum of the echo or the power spectrum of the noise together with the identity of the antenna making the measurement, whether it was in measurement or calibration mode, internal calibration information, time stamps and various telemetry and housekeeping data.

The instrument source packets, together with information about the METOP orbit and information for converting onboard time to UTC, form the level 0 product used by the ASCAT processor.

3 ASCAT MEASUREMENT PROCESS

3.1 Introduction

In this section we model the measurement signal produced by the ASCAT and show how it is related to the normalised radar cross section of the Earth's surface.

The first few sections describe background theory (side-looking radars, the radar equation and pulse compression) before examining the ASCAT measurement process.

3.2 Side Looking Radars

A side looking radar (SLR) consists of a radar transmitter, antenna, receiver and recording equipment mounted on moving platform (usually an aircraft or spacecraft). The radar antenna points sideways, perpendicular to the flight path of the platform, and looks down at the ground. The geometry of this situation is illustrated in Figure 3. As the platform moves, the radar beam illuminates a continuous swath of the terrain. The geometry of the six ASCAT antennas is illustrated in Figure 3.

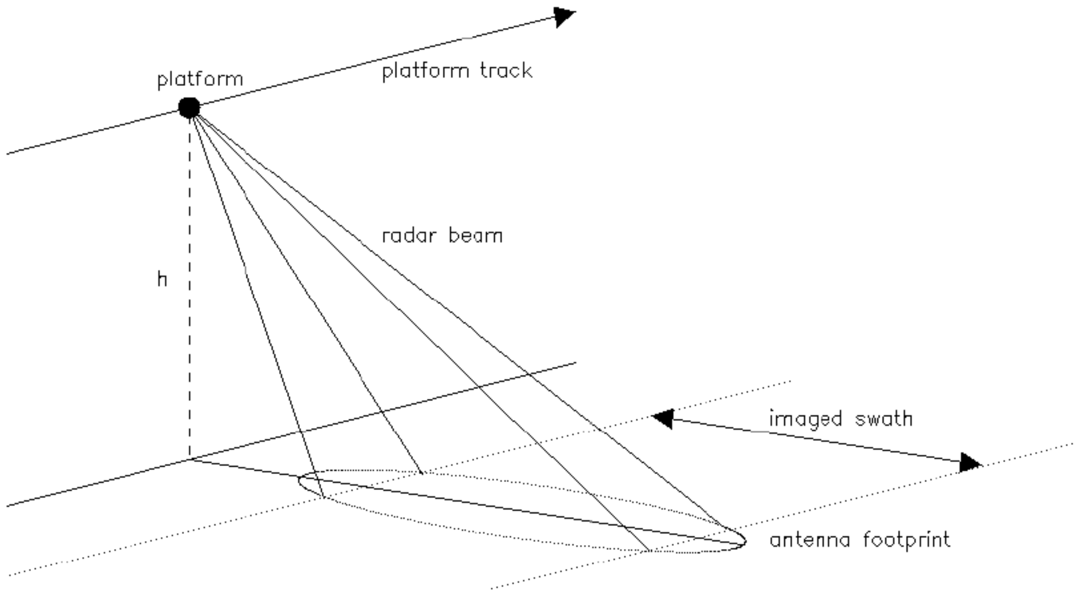

Figure 3: Basic geometry of a side-looking radar.

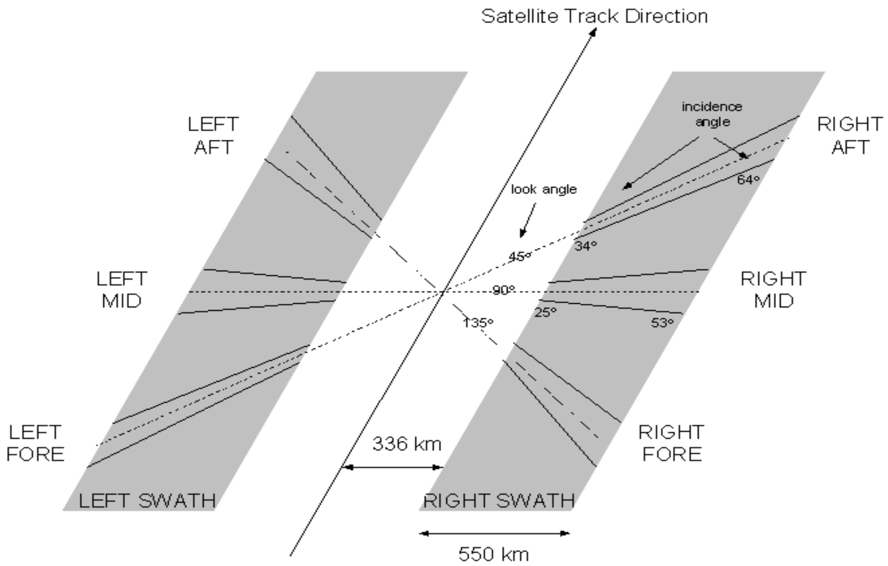

Figure 4: Swath geometry of the ASCAT antennas for a reference orbit height of 822 km.

SLRs usually transmit a sequence of short radar pulses. This allows the discrimination of range, as a radar pulse travelling at the speed of light will take t_1 seconds to travel to an object at range r_1 and back again where $t_1 = 2r_1/c$.

The round trip for a pulse travelling to a second object at range r_2 is $t_2 = 2r_2 / c$ and, provided the radar pulse duration is small in comparison with the difference between t_1 and t_2, it should be possible to discriminate between the two objects.

In a simple SLR which transmits a rectangular shaped pulse of duration τ the resolution in range is as follows:

$$\rho_r = \frac{c\tau}{2}$$ *Equation 1*

This means that objects which are half a pulse length apart in range (and a full pulse duration in terms of round trip delay) can be separated from one another. At a point in the imaged swath at a given ground range r_g (defined as the horizontal distance from the point directly under the radar platform to the point of interest) the angle between the slant range vector r and the vertical will be θ and the resolution in the ground range direction will be this:

$$\rho_g = \frac{c\tau}{2\sin\theta}$$ *Equation 2*

So for an SLR operating at height h, the slant range resolution ρ_r is independent of range, but the resolution on the ground ρ_g will vary across the swath and increase with range.

3.3 The Radar Equation

The radar transmitted power p_t is radiated into free space by the radar antenna. The simplest antenna is an isotropic one which radiates power equally in all directions. In terms of power density (in watts per unit area) the power p_i on an imaginary sphere of radius r for an isotropic antenna is this:

$$p_i = \frac{p_t}{4\pi r^2}$$ *Equation 3*

Most antennas achieve better performance than the isotropic type and the ability to direct radiated power in the direction of a target is measured in terms of the antenna gain G. This may be defined as the ratio of the power directed onto a target using the actual antenna to that which would have resulted from an isotropic antenna. The power density p_d incident on a target from a directive antenna with gain G is then:

$$p_d = \frac{p_t G}{4\pi r^2}$$ *Equation 4*

The incident power is scattered to some extent in all directions. A measure of the proportion of the power scattered back to the illuminating radar is the radar cross section (RCS or σ) and the power density p_e of the echo received at the radar from a target of RCS σ would be as follows:

$$p_e = \frac{p_t G}{4\pi r^2}\frac{\sigma}{4\pi r^2}$$ *Equation 5*

The RCS has units of area and is a characteristic of the target. It can be thought of as a measure of the target's size as seen by the radar. If the radar antenna receives power over an effective area of A_e square meters then the total signal S received by the radar from a target of cross section σ at range r is this:

$$S = \frac{p_t G \sigma A_e}{(4\pi)^2 r^4}$$ *Equation 6*

Antenna theory gives the relationship between the transmitting gain and the receiving effective area of the antenna to be:

$$G = \frac{4\pi A_e}{\lambda^2}$$ *Equation 7*

where λ is the radar wavelength. This allows Equation 6 to be rewritten as follows:

$$S = \frac{p_t G^2 \lambda^2 \sigma}{(4\pi)^3 r^4}$$ *Equation 8*

3.4 Normalised Radar Cross Section

In remote sensing we are interested in measuring the radar backscatter from extended or distributed targets. In this case, the radar cross section is usually put in terms of the radar cross section per unit area, known as normalised radar cross section (NRCS or σ_0), where

$$\sigma = \rho_a \rho_g \sigma_0$$ *Equation 8*

where ρ_a and ρ_g are the radar resolution in the along-track and ground range directions.

3.5 Pulse Compression

The resolution in range for a radar transmitting a sequence of short pulses of duration τ is given in **Equation 1** and in theory we can achieve a fine range resolution by making τ as small as required.

In practice though, particularly at long range, it is not possible to build a transmitter which will provide the high peak power necessary for adequate signal-to-noise performance over such a short pulse, so some other way of obtaining high range resolution must be found.

One solution is to use a long pulse during which the frequency changes in a known manner. In this situation, the echo signal from two separate targets arriving back at the radar cannot be differentiated as a time measurement, but are different in frequency. Signal processing techniques can be used to convert the frequency information into time information, giving an output similar to that from a short high-power pulse. Hence a long frequency modulated (FM) pulse can be “compressed" to give the same information as a short time pulse.

The form of the transmitted pulse can be written as follows:

$$p(t) = \begin{cases} A\sin\left[2\pi\left(f_0 t + \frac{1}{2}\alpha t^2\right)\right] & -\frac{1}{2}\tau < t < \frac{1}{2}\tau \\ 0 & otherwise \end{cases}$$ *Equation 9*

where f_0 is the carrier frequency of the radar and α is the chirp rate. The phase in the above equation is as follows:

$$\phi(t) = 2\pi\left(f_0 t + \frac{1}{2}\alpha t^2\right)$$

Equation 10

which is quadratic in time and the instantaneous frequency across the chirp is given by this equation:

$$f = \frac{1}{2\pi}\frac{d\phi}{dt} = f_0 + \alpha t$$

Equation 11

which is linear in time. When a linear FM pulse has been transmitted and the return echo received by the radar, the signals must undergo pulse compression to obtain the high resolution required. The signal processing involved is known as *de-chirping* or *de-ramping* and can be considered as the removal of the frequency modulation of the returned echo.

The signal processing involved in linear FM compression is known as matched filtering and has the characteristic of maximising the signal to noise ratio of the output signals. In theory, it is possible to find a "matched filter" to compress any transmitted pulse waveform.

The form of a chirp waveform was given earlier and we recast it in complex form as this:

$$p(t) = \begin{cases} Ae^{2\pi i\left(f_0 t + \frac{1}{2}\alpha t^2\right)} & -\frac{1}{2}\tau < t - t_0 < \frac{1}{2}\tau \\ 0 & otherwise \end{cases}$$

Equation 12

The matched filter for this is then:

$$h(t) = mAe^{-2\pi i\left[f_0(t_0 - t) + \frac{1}{2}\alpha(t_0 - t)^2\right]}$$

Equation 13

without loss of generality, we can remove the carrier frequency in both Equation 12 and Equation 13 and shift the time origin so that $t_0 = 0$. In addition, we can choose $m = 1/A$ so that the simplified form of the matched filter is this:

$$h(t) = e^{-\pi i\alpha t^2}$$

Equation 14

and the output from the filter in the presence of an input signal alone is this:

$$
\begin{aligned}
y(t) &= \int p(\tau)\, h(t-\tau)\, d\tau \\
&= \int_{-\tau/2}^{\tau/2} A e^{\pi i \alpha \tau^2} e^{-\pi i \alpha (t-\tau)^2} d\tau \\
&= A e^{-\pi i \alpha t^2} \int_{-\tau/2}^{\tau/2} e^{2\pi i \alpha t \tau} d\tau \\
&= A e^{-\pi i \alpha t^2} \frac{\sin(\pi \alpha t \tau)}{\pi \alpha t}
\end{aligned}
$$

Equation 15

After square law detection this becomes:

$$
|y(t)|^2 = |A|^2 B^2 sinc^2(\pi B t)
$$

Equation 16

where $B = \alpha\tau$. The $sinc^2$ function has a resolution approx $1/B$ in time giving a range resolution of $\rho_r = c/(2B)$.

3.6 The ASCAT Measurement Process

The radio frequency signal transmitted by ASCAT is given by this equation:

$$
s_{tx}^{rf}(t) = \begin{cases} a(t)\, e^{2\pi i \left(f_0 t + \frac{1}{2}\alpha t^2\right)} & -\frac{1}{2}\tau < t < \frac{1}{2}\tau \\ 0 & otherwise \end{cases}
$$

Equation 17

where:

$a(t)$	= the amplitude
f_0	= the carrier frequency
α	= the chirp rate
τ	= the duration of the pulse

The signal received at time t from an elemental area at position (x,y) on the surface of the Earth with range r to the radar is the following:

$$s_{rx}^{rf}(t) = K s_{tx}^{rf}(t - t_d)$$ *Equation 18*

where:

$t_d = 2r/c + 2v_r t/c$	= radar time delay
v_r	= the radial velocity between the radar and the element

and *K,* which determines the amplitude of the signal from the element, is given by:

$$K = \sqrt{\frac{\lambda^2 p_{tx} g_{rx} G^2 \sigma_0}{(4\pi)^3 r^4 L_{tx} L_{rx} e^{2 l_{atm} h_{atm} / \cos\theta}}}$$ *Equation 19*

where:

p_{tx}	= peak power
g_{rx}	= the overall gain
L_{tx}	= the transmit loss
L_{rx}	= the receive loss
G	= the one way antenna gain
σ_0	= the normalised radar cross section of the surface
l_{atm}	= the atmospheric loss
h_{atm}	= the atmospheric height
θ	= the incidence angle
r	= the range

The received signal is down-converted and de-ramped with this equation:

$$s_{rx}^{dr}(t) = s_{rx}^{rf}(t)\, e^{-2\pi i\left[f_1 t + \alpha (t - t_c)^2\right]}$$ *Equation 20:*

where f_1 is the down-conversion frequency and t_c is the time at the centre of the receive window.

The ASCAT receive window is split into *M* overlapping looks. The start time of each look is given by this equation:

$$t_i = t_c - \frac{1}{2}\tau_c + (i-1)(1-\beta)\tau_{rl}$$ *Equation 21*

where

τ_c	= the width of the receive window
τ_{rl}	= the duration of each look
β	= the fractional overlap
$i = 1..M$	= the index of each range look

The signal received in each range look window is Fourier-transformed and the contribution from the elemental area is this:

$$f = \int w(t)\, s_{rx}^{dr}(t)\, e^{-2\pi i \nu t} dt$$ *Equation 22*

where $w(t)$ is a windowing function and ν is the discriminator frequency.

In terms of the transmitted signal this is formulated as follows:

$$f = K \int a(t-t_d)\, w(t)\, e^{2\pi i\left[\alpha(t-t_d)^2 + f_0(t-t_d) - f_1 t - \alpha(t-t_c)^2\right] - 2\pi i \nu t} dt$$ *Equation 23*

which simplifies to:

$$f = K \int a(t-t_d)\, w(t)\, e^{2\pi i t(f_0 - f_1 + 2\alpha t_c - 4\alpha r/c - 2v_r/\lambda - 4\alpha v_r t/c - \nu)} dt$$ *Equation 24*

When the integral is performed and the result is subject to squared modulus detection then the peak for a given slant range will occur when:

$$f_0 - f_1 + 2\alpha t_c - \frac{4\alpha r}{c} - \frac{2v_r}{\lambda} = \nu$$ *Equation 25*

which shows that the power from area elements at different ranges maps into different frequencies. Due to the ASCAT design choice of f_1 with respect to f_0, this equation can be written as follows:

$$f_{offset} - \frac{4\alpha r}{c} - \frac{2v_r}{\lambda} = \nu$$

Equation 26

where f_{offset} has a particular known value for each beam.

The power from the elemental area is given by $|f|^2$ and hence the total power in the i'th range look is this:

$$J(\nu) = \int \int |f(\nu)|^2 dx dy$$

Equation 27

The power from the M range looks are averaged on board ASCAT with:

$$\Omega(\nu) = \frac{\sum_i \rho_i J_i}{\sum_i \rho_i}$$

Equation 28

where the ρ_i are the look-averaging weights. This is the basic measurement produced by ASCAT.

Note: Equation 18 and Equation 19 show that the signal received by ASCAT is proportional to the square root of the NRCS of the ground surface (σ_0). This signal undergoes square law detection in Equation 21 and hence the measurement signal produced by ASCAT is directly proportional to σ_0.

The ASCAT generates a series of pulses with a pulse repetition interval (PRI) of approximately 35 milliseconds. These are distributed to each of the antennas in turn, so each antenna produces a measurement every 6 PRI. To reduce the noise and data rate, an average is calculated at every fourth along-track measurement with this:

$$S(\nu) = \sum_{i=1}^{8} w_i \Omega_i$$

Equation 28

where Ω_i are eight successive along-track measurements from a particular antenna and the weights are $w_i = \{0.05, 0.1, 0.15, 0.2, 0.2, 0.15, 0.1, 0.05\}$. This weighted average is recorded in the instrument source packet (ISP). The time interval between the ISP from a particular antenna is 24 PRI.

4 EXTERNAL CALIBRATION

In the previous section, we modelled the measurement signal produced by ASCAT and showed that it can be written as follows for a surface with constant normalised radar cross-section:

$$\boldsymbol{s = \sigma_0 k}$$ *Equation 29*

where:

s	= measurement signal
σ_0	= NRCS of the surface
k	= function of satellite position, satellite velocity, instrument parameters, atmospheric parameters, antenna gain pattern and pointing

This equation can be rewritten as this:

$$\boldsymbol{\sigma_0 = \frac{s}{k}}$$ *Equation 30*

which shows k as a normalisation factor for converting ASCAT power measurements into calibrated NRCS values.

The satellite position and velocity can be accurately estimated and the instrument parameters are known. The atmospheric attenuation is known for C-band. Hence the normalisation factor k can be calculated if we know the in-flight antenna gain pattern and pointing bias (i.e. the deviation of the antenna pointing away from its nominal value). Both of these are estimated by an external calibration procedure using three ground-based transponders, which represent an independent absolute calibration reference.

As the ASCAT passes over the transponders it switches into calibration mode. When the transponders detect a signal from ASCAT, they transmit a signal of known strength back towards the antenna after a pre-set delay. This ensures that the transponder signal reaches ASCAT after the echoes from the ground, allowing the two to be clearly distinguished.

The magnitude of the transponder signal received by ASCAT depends on the position of the transponder in the gain pattern. The locations of the three transponders are carefully chosen so that the antenna gain patterns are well sampled over the course of the METOP 29-day repeat cycle.

A model of the antenna gain (derived from pre-launch characterisation) and pointing bias is fitted to the set of transponder measurements. This determines the antenna orientation. The residual between the model and the actual gain pattern is then examined to give a correction to the model in order to estimate of the actual in-flight gain pattern.

The corrected pointing bias and gain patterns are then used in the ground processing to calculate the normalisation factors for conversion of the ASCAT measurements into σ_0 values. External calibrations are performed at intervals during the instrument lifetime to correct for long-term drift errors.

5 GROUND PROCESSING

The ground processing of ASCAT data is split into two successive stages that produce the full resolution calibrated backscatter and then the spatially averaged backscatter triplets.

5.1 Estimation and Geolocation of Full-Resolution Calibrated NRCS

In the first stage the instrument measurement and noise source packets are used to estimate the filter shape, mean noise and power-gain (PG) product as functions of time. The filter shape is essentially a rolling average of the noise measurements from all beams, smoothed and then normalised at the calibration frequency. The mean noise is obtained by dividing each noise measurement by the corresponding filter shape then averaging over a range of frequencies. The PG product represents the internal calibration correction and is calculated from the ancillary data (voltages and temperatures) in the source packets.

The measurement data is then corrected according to:

$$s_i^* = \left(\frac{s_i}{\chi_i h_i} - n\right)\frac{1}{\lambda}$$

Equation 31

where:

s_i	= measurements across the swath
h_i	= he filter shape estimates across the swath at the time of the measurement
n	= the mean noise at the time of the measurement
λ	= the PG value at the time of the measurement
χ_i	= the number of range looks used across the swath

This is then converted to absolutely calibrated backscatter with the following:

$$\boldsymbol{\sigma_i^0 = s_i^* k_i}$$

Equation 32

where k_i are the normalisation factors produced from the gain pattern and the viewing geometry at the time of the measurement. The location of each backscatter measurement on the surface of the Earth is calculated along with the corresponding incidence and azimuth angles. See Appendix A for more details.

This is the general algorithm for the first stage of the processing:

```
read the input data (source packets, orbit state vector, gain
patterns, processing and instrument parameters)
calculate the filter shape

  calculate the mean noise
  calculate the PG values
  for each echo source packet
    find the appropriate filter shape, mean noise & PG value
    calculate the normalisation factors
    correct the measurement data and convert to backscatter
    calculate the lat, long, inc & az angles
write the results to an output file
```

It is important to note these two conditions:

- The calculation of normalisation factors is the most computationally intensive part of this algorithm. Rather than doing the calculation for every source packet, it is performed in advance at a number of locations around the orbit and interpolation used to estimate the value at the required time.
- Approximately 10 minutes of data are required to estimate the filter shape with sufficient accuracy and hence the first and last five minutes of any period of processed data will have a poorer estimate of the filter shape and a reduced quality.

5.2 Re-sampling of NRCS and Measurement Noise Estimation

In the second stage of the ground processing, the full resolution calibrated and geolocated backscatter in each beam is spatially-averaged around specified points on the Earth's surface to provide co-located backscatter triplets in each swath. The parameter K_p is also calculated, which is an estimate of the error in the mean backscatter. See Appendix B for more details.

The re-sampling approach is illustrated in Figure 5. The orange plots represent the locations of the full-resolution NRCS values from one beam, the red star is the location where the spatial averaging window is applied, and the yellow stars represent the neighbouring locations where similar spatial averaging windows are centred and applied.

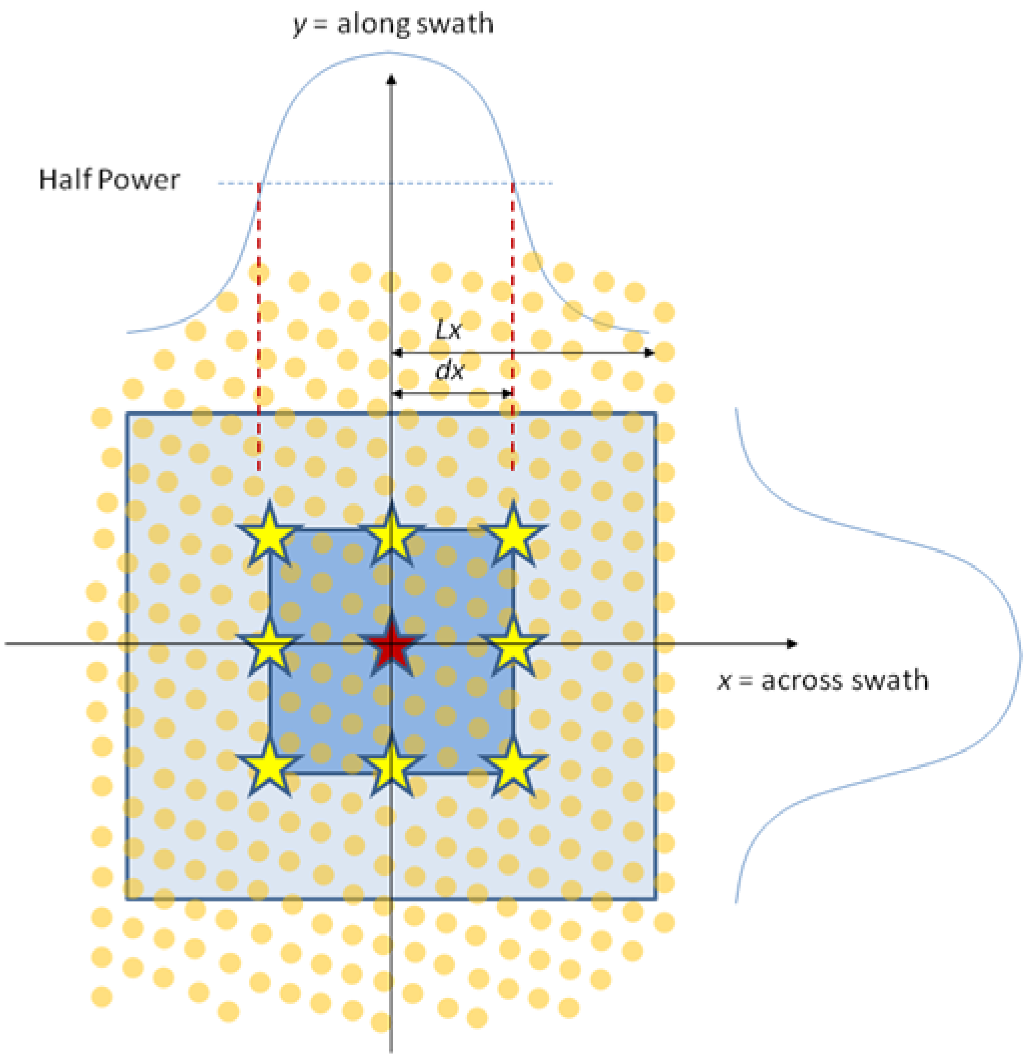

Figure 5: Schematic representation of the ASCAT NRCS re-sampling approach.

Following the heritage from the ERS AMI NRCS processing, the baseline spatial averaging window is a Hamming window with weights given by the following:

$$\boldsymbol{W} = \boldsymbol{F}_x \boldsymbol{F}_y$$

$$F_x = \begin{cases} \alpha + (1-\alpha)\cos\dfrac{\pi x}{L_x} & -L_x < x < L_x \\ 0 & otherwise \end{cases}$$

$$F_y = \begin{cases} \alpha + (1-\alpha)\cos\dfrac{\pi y}{L_y} & -L_y < y < L_y \\ 0 & otherwise \end{cases}$$

Equation 33

where:

α	= 0.54
x	= across-swath horizontal direction
y	= along-swath horizontal direction
L_x	= size of the Hamming window in the across-swath horizontal direction
L_y	= size of the Hamming window in the along-swath horizontal direction

The sampling distances across and along swath *dx* and *dy* are approximately $\frac{1}{2}L_x$ and $\frac{1}{2}L_y$, respectively. The effective resolution of the re-sampled value is defined as the square footprint determined by the half-power of the Hamming window. This is the dark blue area in Figure 5. The actual resolution of the re-sampled value is given by the convolution of the spatial averaging window with the instrument Impulse Response Function (IRF) which determines the footprint of the original NRCS values.

The general algorithm for the second stage of the processing is as follows:

```
read the input data (geolocated and calibrated backscatter)
calculate the grid points in the left hand swath
calculate the grid points in the right hand swath
for each beam
  select the appropriate grid
  for each point in grid
    find the neighbouring sigma0, inc and az angles
    calculate the hamming weights
    calculate the average sigma0 and Kp
    calculate the average inc and az
merge the results from beams 0, 1 & 2 into triplets
merge the results from beams 3, 4 & 5 into triplets
```

6 SUMMARY

This document has examined a variety of issues relating to the ASCAT instrument onboard the METOP series of satellites. Specifically, it has provided the following:

- given an overview of the ASCAT instrument,
- modelled the measurement signal produced by ASCAT,
- described the external calibration process used to estimate the antenna gain patterns and antenna pointing,
- detailed the processing of ASCAT data from raw measurements to calibrated backscatter triplets.

Additionally, algorithms for the geolocation of ASCAT measurements and calculation of K_p are given in the document appendices.

APPENDIX A: THE GEOLOCATION ALGORITHM

Given the position and orientation of an ASCAT antenna with respect to the Earth, we can transform a look direction in the antenna reference frame into the terrestrial reference frame and then find the corresponding point on the surface of the Earth.

Along the main axis of the beam the azimuth look angle is zero and varying the elevation look angle gives a line of points on the Earth surface with a range of approximately 1000 km in the near part of the swath and approximately 1400 km in the far swath.

ASCAT produces a power spectra on a set of discrete frequency values and, as shown in Section 3, each frequency contains the power backscattered from the Earth's surface at a particular range. Geolocation of the measurements is accomplished by finding the points on the centre line of the beam that correspond to the frequencies in the spectra.

The sections that follow will do this:

- Describe the model for the surface of the Earth used in the ASCAT processing,
- introduce the various reference frames used by the METOP satellite and the transformations between them,
- give an overview of how the position, velocity and attitude of the METOP satellite are obtained,
- describe the algorithm for transforming the antenna look direction to find the viewed point on the Earth's surface,
- extend the algorithm so that it finds the point on the centre line of the beam which corresponds to a given frequency in the power spectra.

A.1 EARTH SURFACE MODEL

An ellipsoid is used to model the surface of the Earth. The ellipsoid is an oblate (flattened) spheroid with two different axes: an equatorial radius *a* and a polar radius *b*.

Points on the surface of the ellipsoid satisfy this equation:

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$$

Equation 34

where

a	is the semi-major axis
b	is the semi-minor axis
x	is fixed at 0°
y	is fixed at 90° longitude
z	is directed towards the north pole

The WGS84 reference ellipsoid is used. The parameters of this are as follows:

semi-major axis a	= 6,378,137 m
semi-minor axis b	= 6,356,752 m
flattening	$f = (a - b) / a = 3.35281 \times 10^{-3}$
eccentricity	$e = \sqrt{(a^2 - b^2)/a^2} = 8.1892 \times 10^{-2}$

The vector perpendicular to the surface of the ellipsoid at a point (*x, y, z*) on its surface is given by this:

$$n = x\hat{x} + y\hat{y} + \frac{z}{(1-f)^2}\hat{z}$$

Equation 35

The geodetic latitude ϕ is the angle between the equatorial plane and a line that is normal to the surface of the ellipsoid. Depending on the flattening it may be slightly different from the geocentric latitude which is the angle between the equatorial plane and a line from the centre of the ellipsoid.

The coordinates of a geodetic point are given in geodetic latitude ϕ, longitude λ and the height h of the point over the reference ellipsoid. The rectangular coordinates of the point can be obtained from this:

$$\begin{aligned} x &= (N+h)\cos\phi\cos\lambda \\ y &= (N+h)\cos\phi\sin\lambda \\ z &= \left[\left(1-e^2\right)N+h\right]\sin\phi \end{aligned}$$

Equation 36

where:

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

Equation 37

If *x, y* and *z* lie on the ellipsoid then *h* is zero and these equations can be inverted to determine ϕ and λ as a function of *x*, *y* and *z*. If *x*, *y* and *z* do not lie on the ellipsoid then an approximate solution for ϕ, λ and *h* is given by Maisonobe [RD 1].

A.2 REFERENCE FRAMES

A number of reference frames are used for the METOP satellites.

- Terrestrial reference frame (*T*): This coordinate system is body fixed and rotates with the Earth. Velocities expressed in this system include the rotation of the Earth.
- Local orbital reference frame (*TRL*): The origin of the local orbital reference frame is the satellite centre of mass. The unit vector *L* is in the direction opposite to the Earth centre. The unit vector *T* is normal to the plane defined by *L* and satellite inertial velocity vector *v*. The unit vector *R* completes the right-hand reference frame.
- Local relative orbital reference frame ($T_1R_1L_1$) : This has the same definition as the local orbital reference frame except for the local normal pointing as follows. The unit vector *L* is parallel to the local normal of the Earth's reference ellipsoid directed upward and crossing the space-craft centre of mass.
- Local relative yaw steering orbital reference frame (*T'R 'L '*): This is applicable to the satellite yaw steering mode. It has the same definition as the local orbital reference frame *L* except for the local normal pointing and for its orientation with respect to the spacecraft velocity vector as follows. The vector *L'* is parallel to the local normal of the Earth reference ellipsoid (WGS84). The unit vector *R*' perpendicular to *L'* is in the direction of *v'* which is the velocity vector of the sub-satellite point relative to the Earth model surface (relative ground trace velocity vector).
- Satellite reference frame (*S*): The origin of the satellite reference frame (x_s, y_s, z_s) is on the centre line of the spacecraft at its intersection with the satellite launcher adapter interface plane. The x_s axis coincides with the longitudinal (downward vertical) axis of the satellite on the launch vehicle. The z_s axis coincides with the outward normal to the surface carrying the stowed solar array and the y_s axis forms a right-hand orthogonal set. In orbit, when the satellite is being yaw-steered, the satellite orientation is such that the $-z_s$ axis is aligned close to the downward local normal (nadir), the $-y_s$ axis is aligned close to the ground track velocity direction and the x_s axis completes a right-hand orthogonal set.
- Nominal antenna reference frame (*N*): The origin is the satellite centre of mass. The *z* axis is parallel to the outward normal of the radiating face of the antenna. The *x* axis is parallel to the short sides of the antenna and points roughly towards the Earth. The *y* axis is parallel to the long sides of the antenna and completes the right-hand coordinate system.
- Actual antenna reference frame (*A*): The origin is the satellite centre of mass. The orientation is defined with respect to the *N* coordinate system by consecutive rotations through three Euler angles. First a right-hand rotation through the skew depointing angle ψ around the *z* axis. Second a right-hand rotation through the azimuth depointing angle φ around the resulting *y* axis. Finally a right-hand rotation through the elevation depointing angle θ around the resulting *x* axis.

A.3 TRANSFORMATION BETWEEN REFERENCE FRAMES

Vectors specified in one reference frame can be transformed into another reference frame by multiplying them with the appropriate 3 × 3 transformation matrix.

The transformation matrix T_{A2N} for converting from the actual antenna reference frame A to the nominal antenna reference frame N can be expressed in terms of three angles which measure the difference between the nominal and actual antenna pointing. These angles are determined in the external calibration process by minimising the difference between a set of model and actual transponder gain values.

The transformation matrix T_{S2N} between the spacecraft reference frame S and the nominal antenna coordinate system N, can be determined for each antenna by considering its nominal position with respect to the satellite body.

When the satellite is in yaw steering mode then the satellite reference frame S and the local relative yaw steering frame L' are aligned and the transformation matrix $T_{L'2S}$ is simple to define.

The transformation matrix $T_{L'2T}$ between the relative yaw steering orbital reference plane L' and the terrestrial frame T, can be derived from the satellite orbit state vector and the Earth ellipsoid.

The above transformation matrices can then be used to transform any vector h_a in the actual antenna coordinate system into the terrestrial reference frame by the following:

$$\boldsymbol{h} = \boldsymbol{T}_{L'2T}\boldsymbol{T}_{S2L'}\boldsymbol{T}_{N2S}\boldsymbol{T}_{A2N}\boldsymbol{h}_a \quad \text{Equation 38}$$

A.4 METOP POSITION, VELOCITY AND ATTITUDE

At any given time the state of the METOP satellite can be described by an orbit state vector (OSV) containing the satellite position and velocity.

Every day the Flight Dynamics Facility at EUMETSAT issues the set of predicted times, positions and velocities of the METOP satellite when it crosses the Earth's equatorial plane from the southern to the northern hemisphere (also known as the ascending node crossing, ANX) during the following 72 hours.

Orbit propagation models like the EUMETSAT Proteus model take an ANX OSV as an initial state and use it to estimate the satellite position and velocity at any subsequent time.

The METOP spacecraft has several modes of attitude control. The nominal mode is yaw steering mode (YSM) in which the satellite attitude is aligned with the local relative yaw steering orbital reference frame. The error in the METOP yaw steering is known to be small and, for simplicity, we assume that it is zero–the METOP attitude is assumed to perfectly follow the required yaw steering law.

The spacecraft is normally in yaw steering mode (YSM) but during Out-of-Plane (OOP) manoeuvres. It switches into Fine Pointing Mode (FPM, also known as Geocentric Pointing) for a period of 30 minutes to 40 minutes.

A.5 GEOLOCATION OF ANTENNA LOOK DIRECTION

The relationship between the antenna and the point it views on the surface of the Earth is shown in Figure 6 where *r* is the vector to the point (*x*, *y*, *z*) on the surface of the Earth, *s* is the vector to the position of the satellite (s_x, s_y, s_z) and the unit vector pointing out from the face of the antenna is this:

$$\boldsymbol{h} = \begin{pmatrix} \boldsymbol{h_x} \\ \boldsymbol{h_y} \\ \boldsymbol{h_z} \end{pmatrix}$$ *Equation 39*

Referring to Figure 6 we have:

$$\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{\mu h}$$ *Equation 40*

where μ is the distance between the satellite and the point on the surface of the Earth. As the point point (*x*, *y*, *z*) lies on the Earth ellipsoid, it satisfies the equation:

$$x^2 + y^2 + \frac{z^2}{(1-f_e)^2} = a_e^2$$ *Equation 41*

Combining Equation 40 and Equation 41 leads to the quadratic $a\mu^2 + 2b\mu + c = 0$ where:

$$\begin{aligned} a &= h_x^2 + h_y^2 + \frac{h_z^2}{(1-f_e)^2} \\ b &= s_x h_x + s_y h_y + \frac{s_z h_z}{(1-f_e)^2} \\ c &= s_x^2 + s_y^2 + \frac{s_z^2}{(1-f_e)^2} - a_e^2 \end{aligned}$$ *Equation 42*

This has the two solutions:

$$\mu = \frac{-b \pm \sqrt{b^2 - ac}}{a}$$ *Equation 43*

On physical grounds, we require two real positive values for μ. This is only possible when $b^2 - ac > 0$ and $b < 0$. The smaller of these is the required solution:

$$\mu = \frac{-b - \sqrt{b^2 - ac}}{a}$$ *Equation 44*

Given a normalised-look direction in the antenna reference system:

$$\boldsymbol{h_a} = \begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix}$$ *Equation 45*

we can use Equation 38 to convert it into the terrestrial reference system to give $\boldsymbol{h}$, calculate μ and then use Equation 40 to obtain the corresponding point on the surface of the Earth.

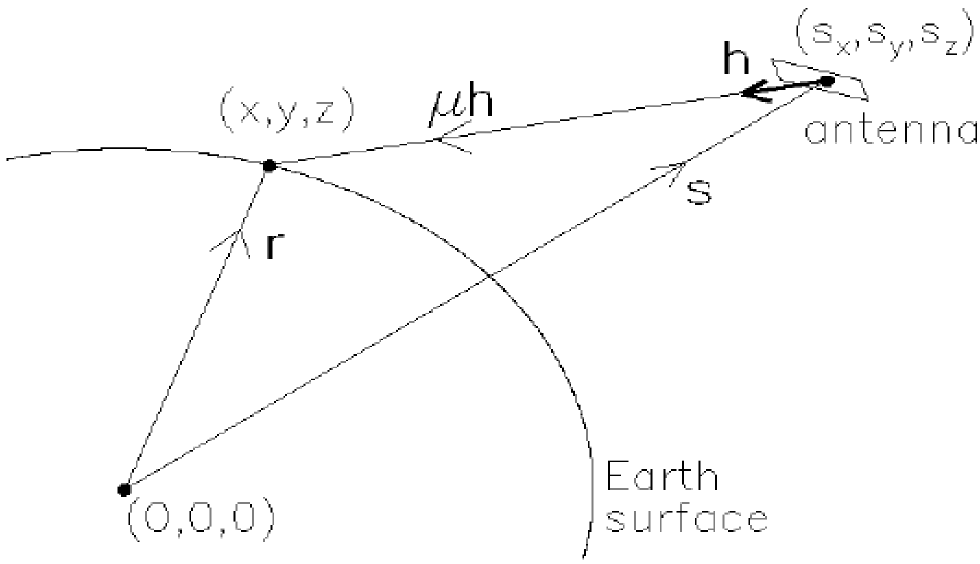

Figure 6: Geometry of an ASCAT antenna viewing the Earth.

A.6 GEOLOCATION OF ASCAT MEASUREMENTS

On the centre line of the beam,the azimuth angle is zero and the vector $\boldsymbol{h_a}$ reduces to this:

$$\boldsymbol{h_a} = \begin{pmatrix} \boldsymbol{x_a} \\ \boldsymbol{0} \\ \sqrt{\boldsymbol{1 - x_a^2}} \end{pmatrix}$$

Equation 46

For a given value of the normalised antenna elevation angle x_a we convert this vector to the terrestrial reference frame, calculate the distance μ between the satellite and point on the Earth surface and then use Equation 26 to calculate the discriminator frequency. This means we can construct the function $f(x_a)$ which gives the discriminator frequency as a function of x_a.

The power spectra produced by ASCAT has the frequencies $v_i = id_f$ where $i = 0...255$ and $d_f = 803$ Hz.

To geolocate each of these values we need to find the value of x_a which satisfies this:

$$f(x_a) = \nu_i$$

Equation 47

This can readily be solved by Newton's method. Finding the value x_a also gives a value for μ which can be used in Equation 40 to give the position on the surface of the Earth. The incidence and azimuth angles for this position can be derived using the Earth ellipsoid and the satellite position and attitude.

Note: Using Newton's method for all the frequencies in the power spectra may be computationally expensive and it may be faster to calculate the function $f(x_a)$ for a set of x_a and then use linear interpolation between the points to find where it has the desired value of v_i.

APPENDIX B: KP ALGORITHM

During the ASCAT processing the backscatter at a point on the Earth's surface is estimated by spatially averaging a number of full resolution backscatter measurements, as described in section 5. The expected error between the spatial average and the true backscatter is also estimated and given in the normalised form:

$$\boldsymbol{K_P} = \frac{\textbf{error in mean backscatter}}{\textbf{mean backscatter}}$$ *Equation 46*

In this appendix we derive expressions for K_p for the following cases:

- The full resolution backscatter measurements are assumed to be independent and the spatial averaging does not involve weighting.
- The full resolution samples are correlated and there is no weighting.
- The full resolution samples are correlated and weighting is used.

The correlations between the full resolution measurements are also derived by examining the processing carried out onboard ASCAT. Pseudo code for the algorithm to calculate K_p is then presented along with a number of faster approximations.

Note: The values of K_p in the nominal ASCAT NRCS products are calculated using the expression given by the last of the three cases.

B.1 INDEPENDENT SAMPLES WITHOUT WEIGHTING

In the simplest case, we assume that the full resolution measurements are independent and that the spatial averaging does not use weighting.

If $x_1, x_2 \dots x_n$ are uncorrelated samples drawn from a distribution, then the sample mean and variance are given by the equation:

$$m = \frac{1}{n}\sum_i x_i$$

$$s^2 = \frac{1}{n-1}\sum_i (x_i - m)^2$$ *Equation 47*

It can readily be shown that *m* and s^2 are unbiased estimates of the distribution mean and variance. It can also readily be shown that the variance of *m* is given by $var(m) = s^2/n$ and hence, we obtain:

$$K_p = \frac{\sqrt{var(m)}}{m} = \frac{s}{m\sqrt{n}}$$

Equation 48

B.2 CORRELATED SAMPLES WITHOUT WEIGHTING

Suppose $x_1, x_2 \ldots x_n$ are correlated samples from a distribution with variance σ^2 and where the correlation between sample x_i and x_j is given by $corr(x_i, x_j) = \rho_{ij}$.

The mean and variance of the samples are given also by Equation 47, repeated here for completeness:

$$m = \frac{1}{n}\sum_i x_i$$

$$s^2 = \frac{1}{n-1}\sum_i (x_i - m)^2$$

Equation 49

The variance of *m* is given by the following:

$$var(m) = var\left(\frac{1}{n}\sum_i x_i\right) = \frac{1}{n^2} var\left(\sum_i x_i\right)$$

Equation 50

Since the variables are correlated this can be written in terms of covariances as this:

$$var(m) = \frac{1}{n^2}\sum_i \sum_j cov(x_i, x_j)$$

Equation 51

and as $cov(x_i, x_j) = \sigma^2 corr(x_i, x_j) = \sigma^2 \rho_{ij}$ this becomes this:

$$var(m) = \frac{\sigma^2}{n^2}\sum_i \sum_j \rho_{ij}$$

Equation 52

The sample variance s^2 can be rewritten as the following:

$$s^2 = \frac{1}{2n(n-1)} \sum_i \sum_j (x_i - x_j)^2$$

Equation 53

If $y_i = x_i - m$ then this becomes:

$$s^2 = \frac{1}{2n(n-1)} \sum_i \sum_j (y_i - y_j)^2$$

Equation 54

and taking the expected values gives us this:

$$E\left[s^2\right] = \frac{1}{2n(n-1)} \sum_i \sum_j \left(E\left[y_i^2\right] - 2E\left[y_i y_j\right] + E\left[y_j^2\right]\right)$$

Equation 55

which becomes the following:

$$E\left[s^2\right] = \frac{1}{2n(n-1)} \sum_i \sum_j \left(\sigma^2 - 2\rho_{ij}\sigma^2 + \sigma^2\right)$$

Equation 56

and reduces to this:

$$E\left[s^2\right] = \frac{\sigma^2}{n(n-1)} \left(n^2 - \sum_i \sum_j \rho_{ij}\right)$$

Equation 57

So the expected sample variance s^2 is not the same as the distribution variance σ^2 but is related to it through the correlation coefficients ρ_{ij}. If these are known and the sample variance is used as an approximation of $E\,[s^2]$ then the above equation allows σ^2 to be determined. Equation 52 then gives $var\,(m)$ and we then obtain $K_p = \sqrt{var(m)}\,/\,m$.

B.3 CORRELATED SAMPLES WITH WEIGHTING

If the samples are weighted, then the sample mean and variance are given by this equation:

$$m = \frac{1}{N} \sum_i w_i x_i$$

$$s^2 = \frac{1}{N} \sum_i w_i \left(x_i - m\right)^2$$

Equation 58

where $N = \Sigma\, w_i$. Following the same procedure as the previous section then gives this result:

$$var\left(m\right) = \frac{\sigma^2}{N^2} \sum_i \sum_j w_i w_j \rho_{ij}$$

Equation 59

and

$$E\left[s^2\right] = \frac{\sigma^2}{N^2} \left(N^2 - \sum_i \sum_j w_i w_j \rho_{ij}\right)$$

Equation 60

If the ρ_{ij} are known, then the above equation gives σ^2 which allows $var\,(m)$ and $K_p = \sqrt{var(m)}\,/\,m$ to be obtained.

B.4 CORRELATION COEFFICIENTS

The measurements produced by ASCAT are correlated in range and azimuth due to the onboard processing.

Range samples are produced by multiplying the returned echo by a weighting function and Fourier-transforming them. The correlation between range sample *a* and range sample *b* is given by this:

$$\rho_{ab} = \left| \frac{\int w(t)\, e^{-2i\pi(f_a - f_b)t} dt}{\int w(t)\, dt} \right|^2$$

Equation 61

where f_a and f_b are the discriminator frequencies of the two samples and $w(t)$ is the range look weighting function. For our discretely-sampled weighting function and Fourier-transform this becomes:

$$\rho_{ab} = \left| \frac{\sum_{k=1}^{n} w_k e^{-2i\pi(a-b)\delta f k \delta t}}{\sum_{k=1}^{n} w_k} \right|^2$$

Equation 62

where $n = 512$ is the number of points used in the on board processing and δ_f and δ_t are the frequency and time step between adjacent points. This simplifies further as the nature of discrete Fourier transforms leads to $\delta_f \delta_t = 1/n$. With the different weighting functions used for side-beams and mid-beams, the range look correlation coefficients are as follows:

$$\rho_{side} = \begin{cases} 1 & |a-b| = 0 \\ 0.081 & |a-b| = 1 \\ 0.027 & |a-b| = 2 \end{cases}$$

Equation 63

and

$$\rho_{mid} = \begin{cases} 1 & |a-b| = 0 \\ 0.019 & |a-b| = 1 \\ 0.015 & |a-b| = 2 \end{cases}$$

Equation 64

An along-track average is produced by ASCAT after every fourth measurement and is an average of the preceding 8 measurements weighted according to $\alpha = \{0.05, 0.10, 0.15, 0.2, 0.2, 0.15, 0.10, 0.05\}$. Neighbouring azimuthal measurements are thus correlated by the following azimuth correlation coefficients:

$$\rho = \frac{\sum_{i=1}^{4} \alpha_i \alpha_{8-i+1}}{\sum_{i=1}^{8} \alpha_i^2} = \frac{1}{3}$$

Equation 65

The correlation between any two measurements ρ_{ij} is given by the product of the correlation in range and the correlation in azimuth. Note that the range correlation decreases rapidly as the range separation increases and the azimuth correlation only extends to neighbouring samples. Hence, to a good approximation, each measurement is only correlated with samples in its immediate neighbourhood.

B.5 ALGORITHM FOR K_P

Assuming the full resolution backscatter and weights in the region of the desired latitude and longitude are stored in the two dimensional arrays $s[ni, nj]$, and $w[ni, nj]$ where ni and nj are the number of samples in range and azimuth then pseudo code to calculate K_p in a *side beam* is:

```
-- array of range correlations in side beam
rx = [ 1, 0.081, 0.027 ]

-- array of azimuth correlations
ry = [ 1, 0.333 ]

-- sum of weights, mean backscatter and variance

n = total( w )
m = total( w*s )/n
v = total( w*(s-m)^2 )/n
-- calculate the sum in the expression for Kp by
-- looping over all the points in the arrays and
-- for each of these loop over neighbouring points
sum = 0
for i = 0 to ni-1
 for j = 0 to nj-1
  for dx = -2 to 2
   for dy = -1 to 1
   i0 = i + dx
   j0 = j + dy
   if i0>=0 and i0<ni and j0>=0 and j0<nj
    r = rx[abs(dx)]*ry[abs(dy)]
    sum = sum + w[i,j]*w[i0,j0]*r

-- calculate Kp
varm = v*sum/( n*n - sum )
```

B.6 FAST APPROXIMATIONS

The algorithm given in the previous section is computationally intensive as it involves processing the points in a 5 × 3 neighbourhood of every point used in calculating the mean backscatter. If calculation of the mean backscatter requires processing N full resolution backscatter values, then calculation of K_p involves processing $15N$ values. So calculation of K_p requires an order of magnitude more processing than calculation of the mean backscatter.

If we assume that the weights in the neighbourhood are approximately constant and ignore edge effects (where the neighbourhood around points at the edge of the spatial averaging window is reduced in size) then the algorithm simplifies to this:

```
n = total(w)
m = total(w*s)/n
v = total( w*(s-m)^2 )/n
sum = 2.03*total(w*w)
varm = v*sum/( n*n - sum )
kp = sqrt(varm)/m
```

If the weighting in uniform with $w_i = 1$ then n is the number of points in the averaging window and the algorithm simplifies to:

```
m = total(s)/n
v = total( (s-m)^2 )/n
varm = v*2.03*n/( n*n - 2.03*n )
kp = sqrt(varm)/m
```

If n is large then this reduces further to this:

```
m = total(s)/n
v = total( (s-m)^2 )/n
varm = 2.03*v/n
kp = sqrt(varm)/m
```

This result implies that taking the correlation of the full resolution measurements into account increases Kp by a factor of approximately $\sqrt{2}$ compared to the simple case when they were assumed to be independent.